MA125-6C, CALCULUS I

Test 4, April 8, 2015

Name (Print last name first): ......Key......

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

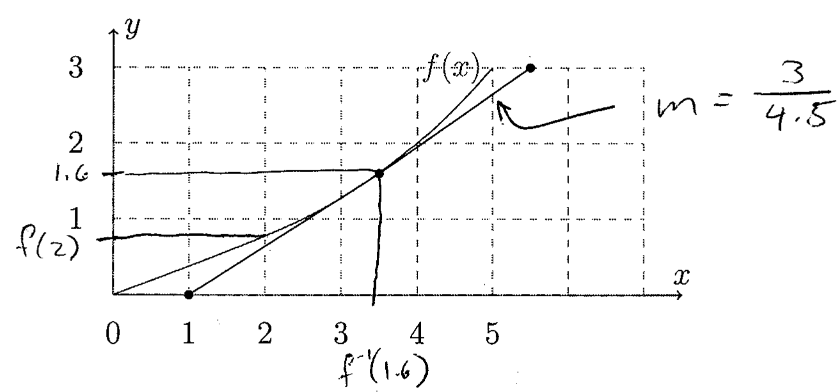
You must simplify your answer when possible.
All problems in Part I are 8 points each.

1. Given the graph of the function $y = f(x)$ below, estimate

   (a) $f(2)$, 0.8

   (b) $f^{-1}(1.6)$, 3.5

   (c) $(f^{-1})'(1.6)$. $(f^{-1})'(1.6) = \frac{1}{f'(f^{-1}(1.6))} = \frac{1}{f'(3.5)} = \frac{1}{3/4.5} = \frac{4.5}{3} = \frac{3}{2}$



2. If $f(x) = \ln(\tan(x))$, find $f'(x)$

$$f'(x) = \frac{1}{\tan(x)} \frac{d}{dx}\left(\tan(x)\right)$$

$$= \frac{\sec^2(x)}{\tan(x)}$$

3. If $f(x) = e^{x^3-x}$, find all critical numbers of $f(x)$ (if any).

$$f'(x) = e^{x^3-x} \frac{d}{dx}\left(x^3-x\right) = e^{x^3-x}\left(3x^2-1\right)$$

$f'(x) = 0$

$3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm \frac{1}{\sqrt{3}}$

4. Evaluate $\displaystyle\int \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{x}{x^2+1}\,dx$$

$$= \int \frac{1}{u}\left(\frac{1}{2}du\right)$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+1| + C$$

5. Solve $e^{x-4} = 5$

$x - 4 = \ln(5)$

$x = \ln(5) + 4$

6. Solve $\ln(x - 4) = 5$,

$x - 4 = e^5$

$x = e^5 + 4$

7. Let $f(x) = x^4 - 3x^2 - 1 = 0$. Compute the second approximate solution x_2, using Newton's method, if the first approximate solution is $x_1 = 2$.

$f'(x) = 4x^3 - 6x$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

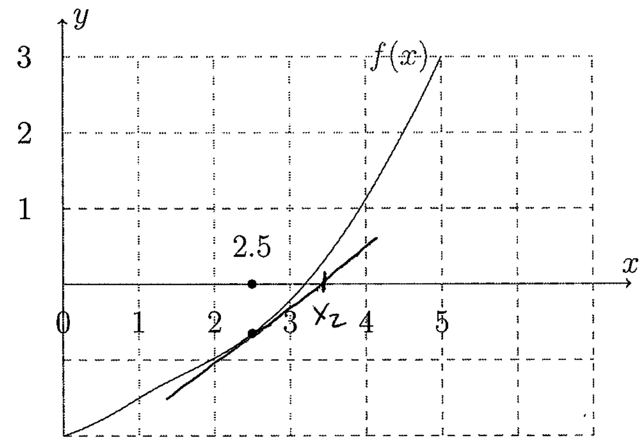
$$= 2 - \frac{(2)^4 - 3(2)^2 - 1}{4(2)^3 - 6(2)}$$

$$= 2 - \frac{16 - 12 - 1}{32 - 12}$$

$$= 2 - \frac{3}{20}$$

$$= \frac{37}{20}$$

8. Use the graph below to draw the location of second approximate solution given that the first approximate solution $x_1 = 2.5$ as indicated.



PART II

1. **[12 points]** Evaluate $\int \frac{\cos(\ln(x^2))}{x}\,dx$

$u = \ln(x^2)$

$du = \frac{1}{x^2}(2x)\,dx$

$= \frac{2}{x}\,dx$

$\frac{1}{2}du = \frac{1}{x}\,dx$

$$\int \frac{\cos(\ln(x^2))}{x}\,dx$$

$$= \int \cos(u)\left(\tfrac{1}{2}\,du\right)$$

$$= \frac{1}{2}\int \cos(u)\,du$$

$$= \frac{1}{2}\sin(u) + C$$

$$= \frac{1}{2}\sin(\ln(x^2)) + C$$

2. **[12 points]** Evaluate $\int e^{5\ln(x)}\,dx$. (Hint: simplify the function before integrating.)

$$\int e^{5\ln(x)}\,dx$$

$$= \int e^{\ln(x^5)}\,dx$$

$$= \int x^5\,dx$$

$$= \frac{1}{6}x^6 + C$$

3. **[12 points]** What are the **absolute** max and min values of the function $f(x) = \ln(x^2 + \frac{1}{2})$ on the interval $[-1, 1]$?

$$f'(x) = \frac{1}{x^2+\frac{1}{2}}(2x) = \frac{2x}{x^2+\frac{1}{2}}$$

Critical Point is $x = 0$

$f(0) = \ln(\frac{1}{2}) < 0$

$f(-1) = \ln(\frac{3}{2}) > 0$

$f(1) = \ln(\frac{3}{2}) > 0$

So $\ln(\frac{3}{2})$ is the abs max & $\ln(\frac{1}{2})$ is the abs min.