MA 125-8B, CALCULUS I

Test 4, November 20, 2014

Name (Print last name first): Key

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 8 points each.

1. Given the graph of the function $y = f(x)$ below, estimate

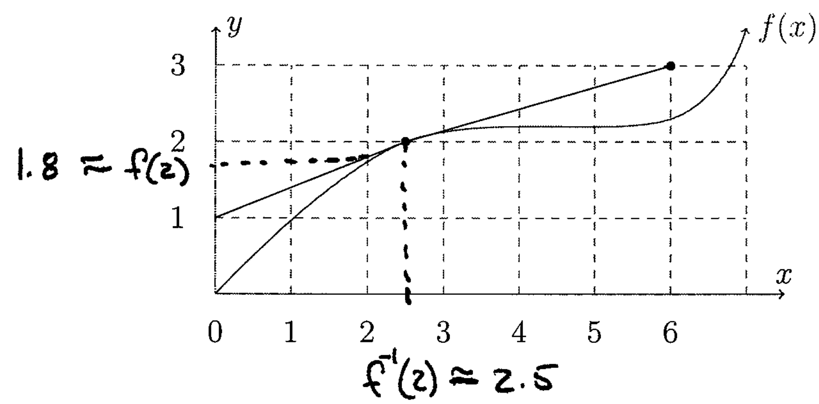
   (a) $f(2)$,

   $$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$$

   (b) $f^{-1}(2)$,

   $$= \frac{1}{f'(2.5)} = \frac{1}{1/3} = 3$$

   (c) $(f^{-1})'(2)$.



2. If $f(x) = \ln(3x^2 - 1)$, find $f'(x)$

$$f'(x) = \frac{1}{3x^2-1}\,\frac{d}{dx}(3x^2-1)$$

$$= \frac{6x}{3x^2-1}$$

3. If $f(x) = xe^{-x^2}$, find all critical numbers of $f(x)$.

$$f'(x) = e^{-x^2} + xe^{-x^2}(-2x) = e^{-x^2} - 2x^2 e^{-x^2} = (1-2x^2)e^{-x^2}$$

$\underline{f'(x)=0}$

$$(1-2x^2)e^{-x^2} = 0$$

$$1-2x^2 = 0$$

$$-2x^2 = -1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm\frac{1}{\sqrt{2}}$$

4. Evaluate $\displaystyle\int \frac{(1+e^{-x})^2}{e^x}\,dx$

$$u = 1+e^{-x}$$

$$du = -e^{-x}\,dx$$

$$-du = \frac{1}{e^x}\,dx$$

$$\int \frac{(1+e^{-x})^2}{e^x}\,dx$$

$$= \int u^2(-du)$$

$$= -\int u^2\,du = -\frac{1}{3}u^3 + C = -\frac{1}{3}(1+e^{-x})^3 + C$$

5. Solve $e^{3x-1} = 5$

$$e^{3x-1} = 5$$

$$\ln\left(e^{3x-1}\right) = \ln(5)$$

$$3x - 1 = \ln(5)$$

$$3x = \ln(5) + 1$$

$$x = \frac{\ln(5) + 1}{3}$$

6. Solve $\ln(3x-1) = 5$,

$$\ln(3x-1) = 5$$

$$e^{\ln(3x-1)} = e^5$$

$$3x - 1 = e^5$$

$$3x = e^5 + 1$$

$$x = \frac{e^5 + 1}{3}$$

7. Let $f(x) = e^x + 2x - 7 = 0$, find two consecutive integers (i.e., find n) so that $f(n) < 0$ and $f(n+1) > 0$. Conclude that there exists a root between n and $n+1$. Use Newton's method, with $x_0 = \frac{2n+1}{2}$ to compute the next approximate solution x_1.

$$f(1) = e + 2 - 7 \approx -2.28 < 0$$

$$f(2) = e^2 + 4 - 7 \approx 4.39 > 0$$

$$x_0 = \frac{2(1)+1}{2} = 1.5$$

$$f'(x) = e^x + 2$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.5 - \frac{f(1.5)}{f'(1.5)}$$

$$\approx 1.5 - \frac{0.481689}{6.48169}$$

$$\approx 1.42568$$

PART II

1. [**12 points**] Evaluate $\int \frac{e^{1/x}}{x^2}\,dx$

$u = \frac{1}{x} = x^{-1}$

$du = -x^{-2}dx = -\frac{1}{x^2}dx$

$-du = \frac{1}{x^2}dx$

$\int \frac{e^{1/x}}{x^2}\,dx$

$= \int e^u(-du)$

$= -\int e^u\,du = -e^u + C = -e^{1/x} + C$

2. [**12 points**] Evaluate $\int_{-e^2}^{-e} \frac{dx}{x\ln|x|}$.

$u = \ln|x|$

$du = \frac{1}{x}dx$

$x = -e,\ u = \ln|-e| = 1$

$x = -e^2,\ u = \ln|-e^2| = 2$

$\int_{-e^2}^{-e} \frac{dx}{x\ln|x|}$

$= \int_2^1 \frac{1}{u}\,du$

$= \left(\ln|u|\right|_2^1 = \ln|1| - \ln|2| = -\ln|2|$

3. **[20 points]** Graph the function $y = f(x) = (x+1)e^x$. Label all x and y intercepts, asymptotes and local/absolute max/min if any. [Hint: use your calculator to estimate $\lim_{x\to-\infty}(x+1)e^x$ by making a table of values; $x=-5$, $x=-10$ and $x=-20$ should suffice.]

x-int

$0 = (x+1)e^x$

$x+1=0$

$x=-1$

y-int

$y=(0+1)e^0$

$=1$

Asymptotes

$\lim_{x\to\infty}(x+1)e^x = \infty$

$\lim_{x\to-\infty}(x+1)e^x = 0$

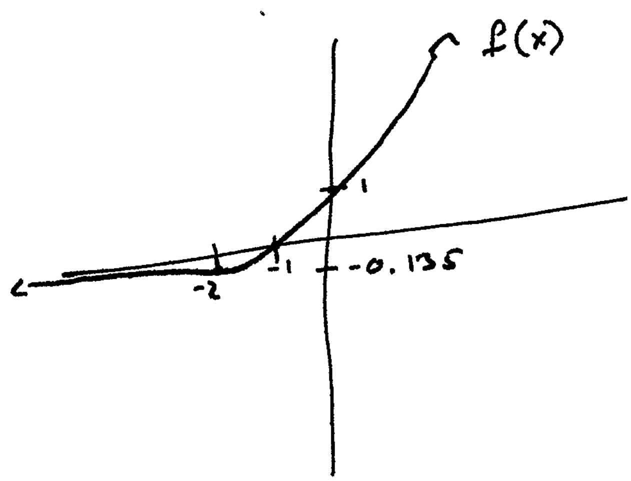
x	$(x+1)e^x$
-5	-0.0269
-10	-0.000408
-20	-3.916×10^{-8}

$f'(x) = (x+1)e^x + e^x$

$= (x+1+1)e^x$

$= (x+2)e^x$

$f'(x)=0$ when $x=-2$



-3 $\quad$ -2 $\quad$ -1

e^x	$+$	$+$
$x+2$	$-$	$+$
$f'(x)$	$-$	$+$

So $x=-2$ is a local & absolute minimum.

Min @ $f(-2) = (-2+1)e^{-2} \approx -0.135$