MA125-8B Chapter 1-2 Practice

Name: Key

Exercise 1. *What is the domain of the function*

$$f(x) = \frac{3}{5x-2}?$$

f is undefined when

$5x - 2 = 0$

$\Rightarrow \quad 5x = 2$

$\Rightarrow \quad x = \frac{2}{5}$

Domain: $(-\infty, \frac{2}{5}) \cup (\frac{2}{5}, \infty)$

Exercise 2. *Determine if the following functions are even, odd, or neither:*

(a) $f(x) = 2x^5 + 6x^2 + 2$

(b) $g(x) = \cos(x)$

(c) $h(x) = x^3 - 5x$.

a) $f(-x) = 2(-x)^5 + 6(-x)^2 + 2$

$= -2x^5 + 6x^2 + 2$

Since $f(-x) \neq f(x)$ & $f(-x) \neq -f(x)$,

f is neither even nor odd.

b) $g(-x) = \cos(-x) = \cos(x) = g(x)$

Even

c) $h(-x) = (-x)^3 - 5(-x)$

$= -x^3 + 5x$

$= -(x^3 - 5x)$

$= -h(x)$

Odd

Exercise 3. *Calculate the limit*

$$\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3}.$$

$$\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3}$$

$$= \lim_{x\to -3} \frac{(x-3)(x+3)}{(x-1)(x+3)}$$

$$= \lim_{x\to -3} \frac{x-3}{x-1}$$

$$= \frac{-3-3}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$$

Exercise 4. *Show the the equation*

$$3x^3 - 4x^2 + x - 1 = 0$$

has a solution between 1 and 2.

Since $f(x) = 3x^3 - 4x^2 + x - 1$ is a polynomial, it is continuous everywhere. Thus, it is continuous on $[1,2]$.

Further, $f(1) = 3(1)^3 - 4(1)^2 + (1) - 1 = -1 < 0$ and

$f(2) = 3(2)^3 - 4(2)^2 + (2) - 1 = 24 - 16 + 2 - 1 = 9 > 0.$

Thus, the Intermediate Value Theorem says that there exists at least one solution in $(1,2)$.

Exercise 5. *Find*

$$\lim_{x\to 2^+} \frac{x}{2x-4} \quad \textit{and} \quad \lim_{x\to 2^-} \frac{x}{2x-4}.$$

$$\lim_{x\to 2^+} \frac{x}{2x-4} = \infty$$

since x near 2 & $x>2$ implies $2x-4$ is near zero but $2x-4>0$.

$$\lim_{x\to 2^-} \frac{x}{2x-4} = -\infty$$

Since x near 2 & $x<2$ implies $2x-4$ is near zero but $2x-4<0$.

Exercise 6. *If the position of an intergalactic spaceship is given by*

$$s(t) = 9999t^3 + 543t,$$

find the instantaneous velocity of the ship at time $t = 15$ *seconds.*

$$v(t) = s'(t) = 9999(3t^2) + 543$$

$$= 29997t^2 + 543$$

$$v(15) = 29997(15)^2 + 543$$

$$= 6749868$$

Exercise 7. *Differentiate the following functions:*

(a) $f(x) = 3x^3 - 4x^2 + 5x - 1$

(b) $g(x) = x^2(1 - 2x)$

(c) $h(x) = 2^{40}$

(d) $r(x) = \frac{3}{x^3} - 10\sin(x)$

a) $f'(x) = 3(3x^2) - 4(2x) + 5(1) + 0$

$= 9x^2 - 8x + 5$

b) $g(x) = x^2(1-2x) = x^2 - 2x^3$

$g'(x) = 2x - 2(3x^2)$

$= 2x - 6x^2$

c) $h'(x) = 0$ since 2^{40} is a constant.

d) $r(x) = \frac{3}{x^3} - 10\sin(x) = 3x^{-3} - 10\sin(x)$

$r'(x) = 3(-3x^{-4}) - 10(\cos(x))$

$= -9x^{-4} - 10\cos(x)$

(e) $u(x) = x^3 \tan(x)$

(f) $m(x) = \frac{\cos(x)}{1-\sin(x)}$

(g) $k(x) = \sin(x\cos(x))$

(h) $v(x) = (4x - 15x^3)^{48}$

e) $u'(x) = x^3 \frac{d}{dx}(\tan x) + \tan(x) \frac{d}{dx}(x^3)$

$= x^3 \sec^2 x + 3x^2 \tan(x)$

f) $m'(x) = \dfrac{(1-\sin x)\cancel{\cos(x)} \frac{d}{dx}(\cos x) - \cos(x) \frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$

$= \dfrac{(1-\sin x)(-\sin x) - \cos(x)(-\cos x)}{(1-\sin x)^2}$

$= \dfrac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} = \dfrac{1-\sin x}{(1-\sin x)^2} = \dfrac{1}{1-\sin x}$

g) $k'(x) = \cos(x\cos(x)) \frac{d}{dx}(x\cos(x))$

$= \cos(x\cos(x)) \left(x \frac{d}{dx}(\cos(x)) + \cos(x) \frac{d}{dx}(x) \right)$

$= \cos(x\cos(x)) (x(-\sin x) + \cos(x)(1))$

$= \cos(x\cos(x)) (\cos(x) - x\sin x)$

h) $v'(x) = 48(4x-15x^3)^{47} \frac{d}{dx}(4x - 15x^3)$

$= 48(4x-15x^3)^{47}(4 - 15(3x^2))$

$= 48(4x-15x^3)^{47}(4 - 45x^2)$