Test 3 MA 125-6A October 28, 2013

Name: Key Signature: ______

**SHOW ALL YOUR WORK!**

If you have time, find a way to check your answers.

## Part 1

1. [5 points] Find the limit: $\lim_{x\to\infty} e^{-x^2}$

$$\lim_{x\to\infty} e^{-x^2} = \lim_{x\to\infty} \frac{1}{e^{x^2}} = 0 \quad \text{since} \quad \lim_{x\to\infty} e^{x^2} = \infty$$

2. [5 points] Evaluate $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[3]{x}}$.

$$\lim_{x\to\infty} \frac{\ln x}{x^{1/3}} \left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{3}x^{-2/3}} = 3\lim_{x\to\infty} \frac{x^{2/3}}{x} = 3\lim_{x\to\infty} \frac{1}{x^{1/3}} = 0$$

3. [5 points] Evaluate $\sin^{-1}\left(\frac{1}{2}\right)$

$\pi/6$

4. [5 points] Find $y'$ if $y = \frac{e^{-x}}{x^2-1}$.

$$y' = \frac{(x^2-1)\frac{d}{dx}(e^{-x}) - e^{-x}\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{-e^{-x}(x^2-1) - e^{-x}(2x)}{(x^2-1)^2}$$

$$= \frac{-e^{-x}(x^2+2x-1)}{(x^2-1)^2}$$

5. [5 points] Evaluate $\lim\limits_{t\to\infty} te^t$

Since $\lim\limits_{t\to\infty} t = \infty$ & $\lim\limits_{t\to\infty} e^t = \infty$, this is not an indeterminate form. Thus,

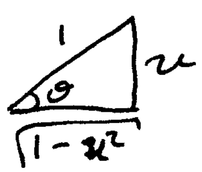
$$\lim_{t\to\infty} te^t = \infty.$$

6. [5 points] Simplify the expression $\tan(\sin^{-1}(u))$

Let $\theta = \sin^{-1} u$. Then $\sin\theta = u$.

Then $\tan(\sin^{-1} u) = \tan\theta$

$$= \frac{u}{\sqrt{1-u^2}}$$



# Part 2

1. [14 points] Evaluate $\lim_{x\to\infty} x^3 e^{-x^2}$

$$\lim_{x\to\infty} x^3 e^{-x^2} \quad (\infty \cdot 0)$$

$$= \lim_{x\to\infty} \frac{x^3}{e^{x^2}} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{3x^2}{2x e^{x^2}}$$

$$= \lim_{x\to\infty} \frac{3x}{2e^{x^2}} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{3}{4x e^{x^2}}$$

$$= 0$$

2. [14 points] Differentiate $f(t) = \cos^{-1}(t^2)$

$$f'(t) = -\frac{1}{\sqrt{1-(t^2)^2}} \frac{d}{dt}(t^2)$$

$$= -\frac{2t}{\sqrt{1-t^4}}$$

3. [14 points] Use logarithmic differentiation to calculate the derivative of

$$y = \frac{x^{\frac{3}{4}}\sqrt{x^2+4}}{(3x+6)^5}$$

$$\ln y = \frac{3}{4}\ln x + \frac{1}{2}\ln(x^2+4) - 5\ln(3x+6)$$

$$\frac{1}{y}y' = \frac{3}{4}\frac{1}{x} + \frac{1}{2}\frac{1}{x^2+4}(2x) - 5\frac{1}{3x+6}(3)$$

$$\frac{1}{y}y' = \frac{3}{4}\frac{1}{x} + \frac{x}{x^2+4} - \frac{15}{3x+6}$$

$$y' = \frac{x^{3/4}\sqrt{x^2+4}}{(3x+6)^5}\left(\frac{3}{4x} + \frac{x}{x^2+4} - \frac{15}{3x+6}\right)$$

4. [14 points]

(10pts) (a) Find the linearization of the function $f(x) = \cos(x)$ at $a = \frac{\pi}{2}$.

$L(x) = f(a) + f'(a)(x-a)$

$f(a) = \cos(\pi/2) = 0$

$f'(x) = -\sin x$

$f'(a) = -\sin(\pi/2) = -1$

So,

$L(x) = -(x - \pi/2)$

$= \pi/2 - x$

(4pts) (b) Use the linearization to estimate $\cos\left(\frac{\pi}{2} + \frac{1}{10}\right)$.

$\cos(\pi/2 + \frac{1}{10}) \approx L(\pi/2 + \frac{1}{10}) = \pi/2 - (\pi/2 + \frac{1}{10}) = -\frac{1}{10}$

5. [14 points] If $f(x) = e^{\cos(x)} + \cos(e^x)$, find $f'(x)$.

$f'(x) = e^{\cos x} \frac{d}{dx}(\cos x) + \left(-\sin(e^x) \frac{d}{dx}(e^x)\right)$

$= -\sin x \; e^{\cos(x)} - e^x \sin e^x$