Test 2A **MA125-6A** **October 2, 2013**

Name: Key **Signature:** ____________

SHOW ALL YOUR WORK!

If you have time, find a way to check your answers.

Part 1

1. [6 points] Find y' if $y = x^6 \sin(x)$.

$$y' = \frac{d}{dx}(x^6)\sin(x) + x^6 \frac{d}{dx}(\sin x)$$
$$= \boxed{6x^5 \sin(x) + x^6 \cos(x)}$$

2. [6 points] Differentiate the function $f(z) = 5z^{40}$.

$$f'(z) = 5\frac{d}{dz}(z^{40}) = 5(40z^{39}) = \boxed{200z^{39}}$$

3. [6 points] Let $v(s) = g(h(s))$, where $h'(0) = -3$, $h(0) = -1$ and $g'(-1) = 4$. Find $v'(0)$.

$$v'(s) = g'(h(s))\,h'(s)$$
$$v'(0) = g'(h(0))\,h'(0)$$
$$= g'(-1)(-3)$$
$$= (4)(-3) = \boxed{-12}$$

4. [6 points] Find the derivative of the function $g(u) = 7u^3 + \frac{-5}{u^2} + 4u - \sqrt{3}$.

$$g'(u) = 7\frac{d}{du}(u^3) - 5\frac{d}{du}(u^{-2}) + 4\frac{d}{du}(u) - \frac{d}{du}(\sqrt{3})$$
$$= 7(3u^2) - 5(-2u^{-3}) + 4(1) - 0$$
$$= \boxed{21u^2 + 10u^{-3} + 4}$$

5. [6 points] Find the derivative of the function $f(x) = (\tan(x))^{30}$.

$$f'(x) = 30(\tan(x))^{29} \frac{d}{dx}(\tan(x))$$

$$= \boxed{30(\tan(x))^{29} \sec^2(x)}$$

6. [6 points] Find the values of x for which the curve $y = 2x^3 + 7x^2 + 4x - 2$ has a horizontal tangent line.

We want to find x so that $y'(x) = 0$.

$$y'(x) = 2(3x^2) + 7(2x) + 4(1) - 0$$

$$= 6x^2 + 14x + 4$$

$$= 2(3x^2 + 7x + 2)$$

$$= 2(3x^2 + 6x + x + 2)$$

$$= 2(3x(x+2) + (x+2))$$

$$= 2(3x+1)(x+2)$$

So, $y'(x) = 0$ when

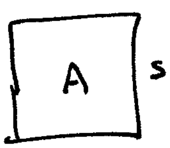
$3x + 1 = 0$ or $x + 2 = 0$

$3x = -1$

$\boxed{x = -\frac{1}{3}}$ $\boxed{x = -2}$

7. [6 points] Each side of the square is increasing at a rate of 1 cm/s. At what rate is the area of the square increasing when the area of the square is 9 cm^2?

Let s be the side length. Let A be the area.



We know

$$\frac{ds}{dt} = 1 \text{ cm/s}$$

We want

$$\left.\frac{dA}{dt}\right|_{s = 3\text{cm}}$$

A & s are related by

$$A = s^2$$

So,

$$\frac{dA}{dt} = 2s\frac{ds}{dt}$$

Thus,

$$\left.\frac{dA}{dt}\right|_{s=3} = 2(3)(1) = \boxed{6 \text{ cm}^2/\text{s}}$$

Part 2

1. [7 points] Find the equation of the tangent line to the parabola $y = x^2 - 7x + 9$ at the point (3,-3)

Since, $y' = 2x - 7$, the slope of the tangent line when $x = 3$ is $y'(3) = 2(3) - 7 = -1$. Thus, the equation for the tangent line is

$$y + 3 = -1(x-3)$$

$$y = -x + 3 - 3$$

$$\boxed{y = -x}$$

2. [10 points] Let $f(x) = x^2 + 3$. Use the **limit definition of the derivative** to find the derivative $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 3 - (x^2 + 3)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0} 2x + h = \boxed{2x}$$

3. [11 points] Use implicit differentiation to find the derivative $\frac{dy}{dx}$ if $y^3 = \sin(xy)$.

$$\frac{d}{dx}(y^3) = \frac{d}{dx}(\sin(xy))$$

$$3y^2 \frac{dy}{dx} = \cos(xy)\frac{d}{dx}(xy)$$

$$3y^2 \frac{dy}{dx} = \cos(xy)\left(y + x\frac{dy}{dx}\right)$$

$$3y^2 \frac{dy}{dx} = y\cos(xy) + x\cos(xy)\frac{dy}{dx}$$

$$3y^2 \frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$$

$$\frac{dy}{dx}\left(3y^2 - x\cos(xy)\right) = y\cos(xy)$$

$$\boxed{\frac{dy}{dx} = \frac{y\cos(xy)}{3y^2 - x\cos(xy)}}$$

4. [18 points] If $f(x) = \dfrac{x^2}{1+x}$, find $f''(1)$.

$$f'(x) = \frac{(1+x)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$= \frac{(1+x)(2x) - x^2(1)}{(1+x)^2}$$

$$= \frac{2x + 2x^2 - x^2}{(1+x)^2}$$

$$= \frac{x^2 + 2x}{(1+x)^2}$$

$$f''(x) = \frac{(1+x)^2\frac{d}{dx}(x^2+2x) - (x^2+2x)\frac{d}{dx}((1+x)^2)}{(1+x)^4}$$

$$= \frac{(1+x)^2(2x+2) - (x^2+2x)(2(1+x)(1))}{(1+x)^4}$$

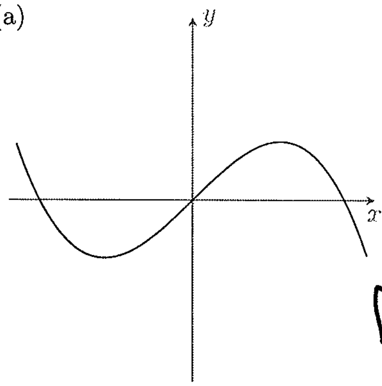
$$= \frac{2(1+x)^3 - 2x(x+2)(1+x)}{(1+x)^4}$$

$$f''(1) = \frac{2(2)^3 - 2(3)(2)}{2^4}$$

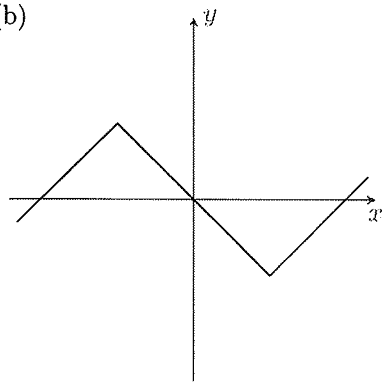
$$= \frac{16-12}{16} = \frac{4}{16} = \boxed{\frac{1}{4}}$$

5. [12 points] Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV.

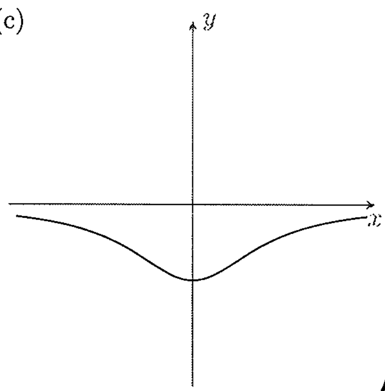
(a)



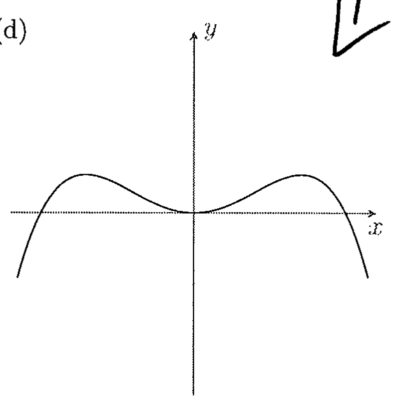
(b)



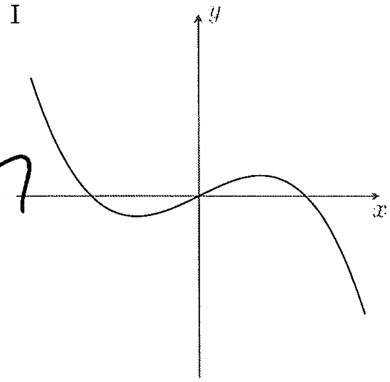
(c)



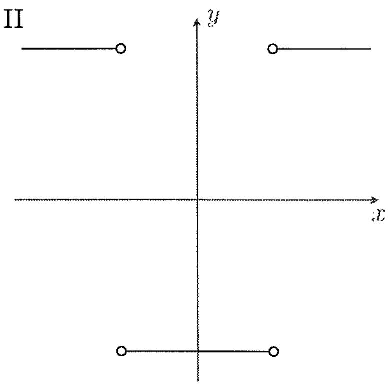
(d)



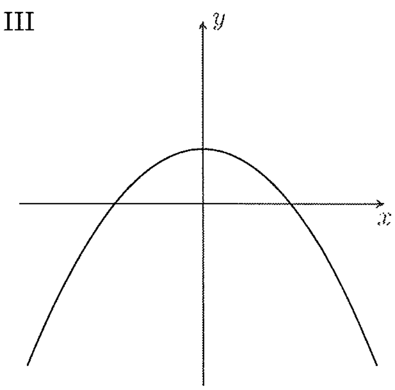
I



II



III



IV