Test 1 MA 125-6A September 11, 2013

Name: Key Signature: ____________

SHOW ALL YOUR WORK!

If you have time, find a way to check your answers.

Part 1

1. [7 points] Evaluate $\lim_{s\to 0} \frac{\sqrt{4+s}-2}{s}$

$\lim_{s\to 0} \frac{\sqrt{4+s}-2}{s}$

$= \lim_{s\to 0} \frac{\sqrt{4+s}-2}{s}\left(\frac{\sqrt{4+s}+2}{\sqrt{4+s}+2}\right)$

$= \lim_{s\to 0} \frac{4+s-4}{s(\sqrt{4+s}+2)}$

$= \lim_{s\to 0} \frac{s}{s(\sqrt{4+s}+2)}$

$= \lim_{s\to 0} \frac{1}{\sqrt{4+s}+2}$

$= \frac{1}{\sqrt{4}+2}$

$= \boxed{\frac{1}{4}}$

2. [7 points] Given that $\lim_{u\to a} f(u) = -7$ and $\lim_{u\to a} g(u) = -6$, find

$$\lim_{u\to a} \frac{f(u)}{-g(u)}$$

$\lim_{u\to a} \frac{f(u)}{-g(u)}$

$= \frac{\lim_{u\to a} f(u)}{\lim_{u\to a} -g(u)}$

$= \frac{\lim_{u\to a} f(u)}{-\lim_{u\to a} g(u)}$

$= \frac{-7}{-(-6)}$

$= \boxed{-\frac{7}{6}}$

3. [7 points] Use the definition of continuity to evaluate [note that your answer must be a number]

$$\lim_{s\to 0} \sin(s+\sin(s))$$

Since sin is continuous on $(-\infty, \infty)$,

$\lim_{s\to 0} \sin(s+\sin(s))$

$= \sin\left(\lim_{s\to 0} (s+\sin(s))\right)$

$= \sin\left(\lim_{s\to 0} s + \lim_{s\to 0} \sin(s)\right)$

$= \sin(0)$

$= \boxed{0}$

4. [7 points] Evaluate $\lim_{w\to 0} \frac{\sin(6w)}{2w}$

$$\lim_{w\to 0} \frac{\sin(6w)}{2w}$$

$$= \lim_{w\to 0} \frac{3}{3} \cdot \frac{\sin(6w)}{2w}$$

$$= 3 \lim_{w\to 0} \frac{\sin(6w)}{6w}$$

$$= 3 \cdot 1 = \boxed{3}$$

5. [7 points] Evaluate $\lim_{s\to 5} \frac{s^2-2s-15}{s-5}$

$$\lim_{s\to 5} \frac{s^2-2s-15}{s-5}$$

$$= \lim_{s\to 5} \frac{(s-5)(s+3)}{s-5}$$

$$= \lim_{s\to 5} s+3 = \boxed{8}$$

6. [7 points] Determine the x-values where the following function $f(x) = \frac{x^2-1}{x^2+7x+12}$ **fails** to be continuous.

f fails to be continuous where it is undefined. That is, where $x^2+7x+12=0$.

Solving this gives $\boxed{x=-3 \;\&\; x=-4.}$

$$(x+3)(x+4)=0$$

$$x+3=0 \qquad x+4=0$$

$$x=-3 \qquad x=-4$$

Part 2

1. [12 points] Evaluate $\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 + 6x + 5}$

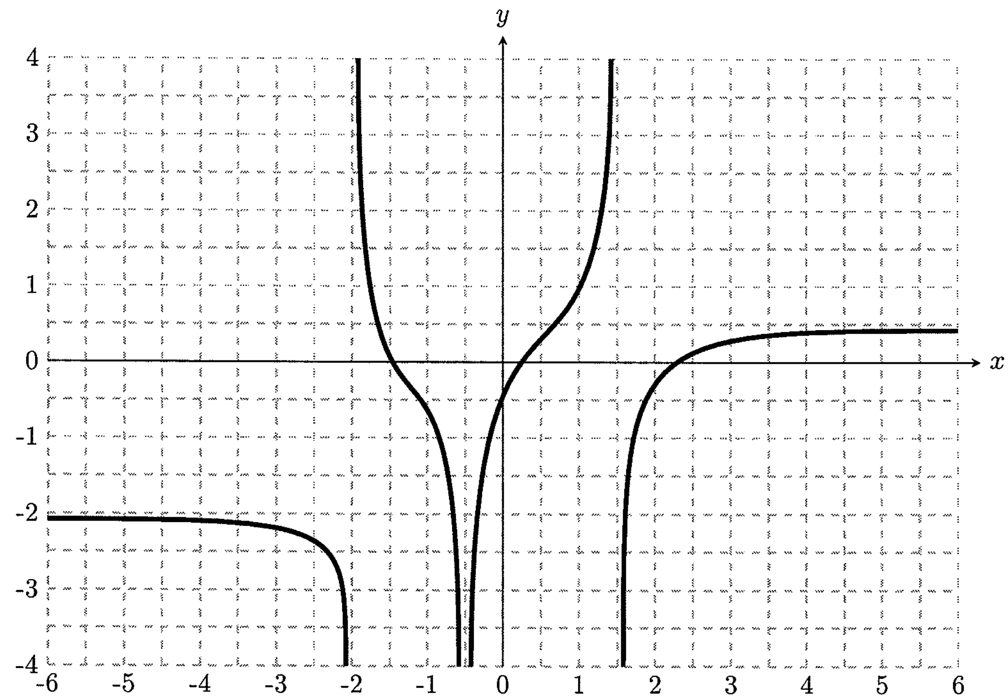
$$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 + 6x + 5}$$

$$= \lim_{x \to -1} \frac{(x-2)(x+1)}{(x+1)(x+5)}$$

$$= \lim_{x \to -1} \frac{x-2}{x+5}$$

$$= \frac{-1-2}{-1+5} = \boxed{\frac{-3}{4}}$$

2. [8 points] Given the following graph:



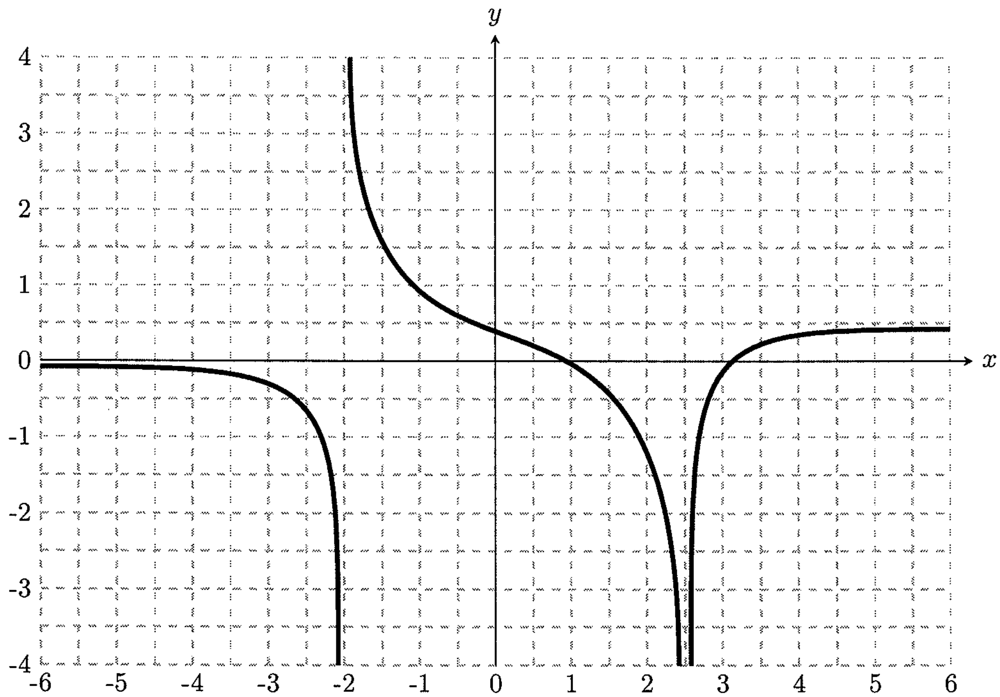
4pts (a) Find all vertical asymptotes (if any):

$\boxed{x = -2, \quad x = -0.5, \quad x = 1.5}$

4pts (b) Find all horizontal asymptotes (if any):

$\boxed{y = -2, \quad y = 0.5}$

3. [8 points] Given the following graph:



4pts (a) Find all vertical asymptotes (if any):

$\boxed{x = -2 \ , \ x = 2.5}$

4pts (b) Find all horizontal asymptotes (if any):

$\boxed{y = 0, \ y = 0.5}$

4. [13 points] Consider the function

$$f(x) = \begin{cases} x+1 & \text{for } x < 4 \\ 21 - x^2 & \text{for } x \geq 4 \end{cases}$$

4pts (a) Evaluate

$$\lim_{x\to 4^-} f(x)$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} x+1 = \boxed{5}$$

4pts (b) Evaluate

$$\lim_{x\to 4^+} f(x)$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} 21 - x^2 = 21-(4)^2 = 21-16 = \boxed{5}$$

5pts (c) Is this function continuous at $x = 4$? (Justify your answer)

Yes, since $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x) = 5$, then $\lim_{x\to 4} f(x) = 5$.

Furthermore, $\lim_{x\to 4} f(x) = 5 = f(4)$. Thus, f is continuous at $x=4$.

5. [17 points] Given the function $f(x) = \dfrac{x^2-4}{x^2-1}$ determine:

3pts (a) the domain of f.

Solving $x^2-1=0$

gives $x=1$, & $x=-1$.

$x^2-1=0$

$(x-1)(x+1)=0$

$x-1=0 \quad x+1=0$

$x=1 \quad x=-1$

The domain is all x such that $x^2-1 \neq 0$. Thus, the domain is

$\boxed{(-\infty,-1)\cup(-1,1)\cup(1,\infty).}$

3pts (b) the x and y intercepts, if any.

y-intercept

$f(0) = \dfrac{-4}{-1} = 4$

So, y-intercept is at 4

x-intercept

$0 = \dfrac{x^2-4}{x^2-1} = \dfrac{(x-2)(x+2)}{x^2-1}$

so $x=2$ or $x=-2$.

x-intercept at 2 & -2

4pts (c) The vertical asymptotes, if any.

Since $\lim\limits_{x\to 1^+} f(x) = \infty$ & $\lim\limits_{x\to -1^-} f(x) = \infty$, there are vertical asymptotes at $x=1$ & $x=-1$.

4pts (d) the horizontal asymptotes, if any (show work here to justify your answer).

$$\lim_{x\to\infty} \frac{x^2-4}{x^2-1} = \lim_{x\to\infty}\left(\frac{x^2-4}{x^2-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) = \lim_{x\to\infty} \frac{1-\frac{4}{x^2}}{1-\frac{1}{x^2}} = 1$$

$$\lim_{x\to-\infty} \frac{x^2-4}{x^2-1} = \lim_{x\to-\infty}\left(\frac{x^2-4}{x^2-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) = \lim_{x\to-\infty} \frac{1-\frac{4}{x^2}}{1-\frac{1}{x^2}} = 1$$

Thus, there is a horizontal asymptote at $y=1$.

3pts (e) Determine if the function is even, odd, or neither.

$$f(-x) = \frac{(-x)^2-4}{(-x)^2-1} = \frac{x^2-4}{x^2-1} = f(x)$$

Thus, f is even.